

The effect of a topological gauge field on Bose-Einstein condensation

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May 20, 1999

Abstract

We show that Bose-Einstein condensation of charged scalar fields interacting with a topological gauge field at finite temperature is inhibited except for special values of the topological field. We also show that fermions interacting with this topological gauge field can condense for some values of the gauge field.

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1 Introduction

Bose-Einstein condensation (BEC) has attracted much attention since its proposal by Einstein [1], after the fundamental work of Bose [2] on the statistics of integer spin particles. Its first realization was given by London in a model for the Helium superfluidity [3]. More recently, the interest for this phenomenon has increased because of the experimental results on atomic traps and the direct observation of Bose-Einstein condensation (BEC) on gases [4] signaling for a large number of applications. In view of these aspects, it has been a subject closely related to condensed matter physics. However, this situation has changed over the past two decades since the work of Haber and Weldon [5] and Kapusta [6] where BEC was related to spontaneous symmetry breaking in relativistic quantum field theory at finite temperature and density. Long ago, it was shown for massive bosons in N space dimensions and in the absence of external fields that BEC can only occur¹ if $N \geq 3$. This result was first shown in a nonrelativistic context [7] and later generalized to the relativistic case [5,6]. Recently, the extension of these results to curved space-times and the inclusion of non-uniform magnetic fields in the relativistic context were also done [8, 9].

The topology of finite temperature field theory is nontrivial due to the compactification of the time coordinate (after a Euclidean rotation) to a circle of length $\beta = T^{-1}$. This topology and the periodicity of bosonic fields at finite temperature imply that even being constant and uniform the time component of the gauge field A_0 cannot be gauged away from the theory, but just be reduced to a value in the interval $(0, 2\pi/\beta)$, except in the case where $A_0 = 2\pi n/\beta$ (n integer) for which it can be reduced to zero. The relevance of A_0 is manifold. For instance, it became clear after the work of Polyakov [10] and Susskind [11] that the problem of confinement in gauge theories at finite temperatures, which in general can be inferred from the expected value of the Wilson line $\langle \exp i \int dx_\mu A^\mu \rangle$, can be analyzed simply looking at $L = \langle \exp \int_0^\beta d\tau A_0 \rangle$. If $L = 0$ then the charge will be confined and in the other case it is deconfined (see also [12-13]). In general, A_0 is a function of space-time coordinates (τ, \vec{x}) but the most simple case where an effective potential $V(A_0)$ can be calculated is the case of a constant and uniform A_0 as has been

¹For massless bosons this condition reduces to $N \geq 2$, although in this case the critical temperature is infinite.

discussed in Refs. [13-15] (of course for slowly varying A_0 this can be viewed as a good approximation). In particular, scalar and spinor electrodynamics at finite temperature in $N + 1$ space-time dimensions with $A_0 \approx \text{constant}$ have been studied by Actor [15] where the role of A_0 as a “thermal interaction” has been stressed. He also considered abelian gauge theories in the presence of constant A_0 at finite temperature and density, which means a nonzero chemical potential. Recently, in 2+1 dimensional physics it was shown that the presence of an A_0 is essential to keep gauge invariance of the Chern-Simons term at finite temperature [16]. However, the influence of such a gauge field (constant and uniform A_0) in BEC of a gas of relativistic charged bosons has not been investigated.

Here, we present a relativistic quantum field theory approach to BEC of charged scalar fields interacting with a topological gauge field A_μ in $N + 1$ space-time dimensions at finite temperature and density. For calculational purposes we take $A_i = 0$ and $A_0 \approx \text{constant}$ following the lines of Refs. [13-15]. The case $A_i \neq 0$ will be discussed in a separate paper. We show that BEC is inhibited except for special values of A_0 . Moreover, we also show that such a gauge field interacting with a gas of relativistic charged fermions can induce condensation analogous to BEC for some other values of the gauge field. These results are also valid in nonrelativistic context which one can obtain from our relativistic results as a particular limit.

2 Bose-Einstein Condensation

The interaction of charged scalar fields with A_0 is equivalent to modifying the usual periodic (bosonic) boundary conditions for the charged scalar fields at finite temperature to generalized boundary conditions [15, 17, 18] or to the introduction of an imaginary part of the chemical potential (this will be made explicit below). An imaginary chemical potential has been considered recently in the literature to discuss the critical exponents of the Gross-Neveu model in 2+1 dimensions at fixed fermion number [19] and the potential for the phase of the Wilson line for an $SU(N)$ gauge theory at nonzero quark density [20]. In these two papers it was suggested that an imaginary part of the chemical potential allow fermions to condense. In the next section, we are going to show this result explicitly for a gas of relativistic charged fermions.

Let us start writing the partition function for the charged scalar fields Φ and Φ^* with

mass M at finite temperature and density interacting with the topological gauge field $A_\mu = (A_0, \vec{0})$ in $N + 1$ space-time dimensions as (see [6, 15] for details)

$$\mathcal{Z} = \int_{\text{periodic}} D\Phi D\Phi^* \exp \left\{ - \int_0^\beta d\tau \int d^N x \Phi^* [(\partial_0 + A_0 + i\mu)^2 - \partial_i^2 + M^2] \Phi \right\} \quad (1)$$

where μ is the chemical potential and Latin letters indicate spatial coordinates ($i = 1, \dots, N$).

As it is well known, one can express this partition function as a determinant namely,

$$\mathcal{Z}_{\text{Bosons}} = \left\{ \det [-(\partial_0 + A_0 + i\mu)^2 - (\partial_i)^2 + M^2] \Big|_P \right\}^{-1}. \quad (2)$$

The label P means that the eigenvalues of this operator are subjected to periodic boundary conditions and hence they are given by

$$\lambda_{nk} = (\omega_n + A_0 + i\mu)^2 + \vec{k}^2 + M^2, \quad (3)$$

where $\omega_n = 2n\pi/\beta$, with $n \in \mathbb{Z}$, are the Matsubara frequencies for bosonic fields and $\vec{k} \in \mathbb{R}^N$. The above determinant can be calculated by different methods as for example the zeta function [21-23] using

$$\det \mathcal{O} = \exp [(-\partial/\partial s)\zeta(s, \mathcal{O})]_{s=0}, \quad (4)$$

where the generalized zeta function is given by

$$\zeta(s, \mathcal{O}) = \sum_n \lambda_n^{-s}, \quad (5)$$

and λ_n are the eigenvalues of the operator \mathcal{O} . An analytical extension of $\zeta(s, \mathcal{O})$ for the whole s -complex plane is assumed. Then, it can be shown that the free energy $\Omega = -(1/\beta) \ln \mathcal{Z}$ corresponding to the partition function (1) can be written as² [15]

$$\Omega(\beta, \mu, A_0) = -4V \sum_{n=1}^{+\infty} \cos(n\beta A_0) \cosh(n\beta\mu) \left(\frac{M}{2\pi n\beta} \right)^{\frac{N+1}{2}} K_{\frac{1}{2}(N+1)}(n\beta M), \quad (6)$$

where V is the volume in N space dimensions and $K_\nu(x)$ is the modified Bessel function of the second kind. Further, the charge density is given by:

$$\rho \equiv \frac{1}{\beta V} \frac{\partial}{\partial \mu} \ln \mathcal{Z} = -\frac{1}{V} \left(\frac{\partial \Omega}{\partial \mu} \right)_{V,T}. \quad (7)$$

²Note that in this formula, only the real part of the free energy $\Omega(\beta, \mu, A_0)$ is expressed, according to the prescription of introducing the chemical potential as an imaginary time-component gauge potential.

We can use the above free energy to calculate the critical temperatures, densities and dimensions for BEC. Let us illustrate this with the simplest cases where we take $A_0 = 0$ and the limits of ultrarelativistic or nonrelativistic regimes. The general case $A_0 \neq 0$ will be discussed bellow. Let us first consider the ultrarelativistic case. In this situation where the energies involved are much higher than the mass scale M we should take the limit $\beta M \ll 1$, which is easily recognized as a high temperature limit. The condensation condition $\mu \rightarrow M$ also implies $\beta\mu \ll 1$, so that the charge density reads (see the Appendix)

$$\rho = \frac{2\mu}{\pi^{(N+1)/2}} \Gamma\left(\frac{N+1}{2}\right) \zeta(N-1) T^{N-1}, \quad (8)$$

where $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ is the usual Riemann zeta function. Using the fact that the condensate is reached when $\mu = M$, in $N = 3$ space dimensions, we find the critical temperature:

$$T_c = \left(\frac{3\rho}{M}\right)^{1/2}, \quad (9)$$

which coincides with the ultrarelativistic BEC results known in the literature [5-9]. Now, let us discuss briefly the situation for the nonrelativistic limit which here means to take the low temperature limit, $\beta M \gg 1$. At the condensate $\mu = M$, the charge density is given by

$$\rho = \zeta\left(\frac{N}{2}\right) \left(\frac{T_c M}{2\pi}\right)^{\frac{N}{2}}, \quad (10)$$

which for $N = 3$ space dimensions leads to the critical temperature

$$T_c = \frac{2\pi}{M} \left(\frac{\rho}{\zeta(3/2)}\right)^{2/3}, \quad (11)$$

which is in agreement with the well known nonrelativistic BEC results [7-9]. Looking at the Eqs. (8) and (10) for the charge density in N space dimensions for the ultrarelativistic and nonrelativistic cases one can see that the condensate is not defined in two space dimensions for both cases, since the Riemann zeta function $\zeta(s)$ has a pole at $s = 1$, so that condensation occurs for $N > 2$.

Let us now discuss the general case $A_0 \neq 0$. To see explicitly when the condensation occurs, let us rewrite the free energy (6) as

$$\Omega(\beta, \mu, A_0) = \frac{TV}{(2\pi)^N} \int d^N k \Re \left\{ \ln \left[1 - e^{i\beta A_0} e^{-\beta(\omega - \mu)} \right] + (\mu \rightarrow -\mu) \right\}$$

$$= \frac{TV}{2(2\pi)^N} \int d^N k \left\{ \ln \left[1 + e^{-2\beta(\omega-\mu)} - 2 \cos(\beta A_0) e^{-\beta(\omega-\mu)} \right] + (\mu \rightarrow -\mu) \right\} , \quad (12)$$

where $\omega = \sqrt{\vec{k}^2 + M^2}$. The condensation condition is given by the divergence in the free energy or equivalently in the charge density. As the condensation occurs near the zero momenta ($\vec{k} = \vec{0}$) state, this implies that $\omega \rightarrow M$ and the condensation condition is given by

$$1 + e^{-2\beta(M-\mu)} - 2 \cos(\beta A_0) e^{-\beta(M-\mu)} = 0 , \quad (13)$$

or simply

$$\cosh \beta(\mu - M) = \cos \beta A_0 , \quad (14)$$

which can only be satisfied for finite temperatures if $\mu = M$ and

$$A_0 = \frac{2n\pi}{\beta} \quad (n \in \mathbb{Z}) \quad (15)$$

simultaneously. Note that these values for A_0 are precisely the ones that can be gauged to zero. Hence, we can say that BEC is completely suppressed for $A_0 \neq 0$. The above condition on A_0 coincides with the minima of the free energy Eq. (6) indicating that the values where condensation occurs are minima for interaction energy of the charged scalar fields with A_0 . This condition on the topological field A_0 also implies that the trivial ($n = 0$) topological sector is equivalent to the zero temperature limit ($\beta \rightarrow \infty$), as expected.

As a final remark on BEC of charged bosonic particles, let us mention that the divergence condition given by Eqs. (13) and (14) comes from the branch cut of the free energy (12) defined by $\mu_{\text{cut}}^2 \leq M^2$, when A_0 satisfies Eq. (15). The physical region for the chemical potential is then $\mu^2 > M^2$ and the limit $\mu \rightarrow M$ defines the critical point for which BEC happens.

3 Fermions

Let us now show that the above argument for bosonic scalar fields applied to fermionic fields implies that for special values of the topological gauge field A_0 fermions can condense. The partition function in this case can be written as

$$\mathcal{Z}^F = \int_{\text{antiper.}} D\bar{\Psi} D\Psi \exp \left\{ \int_0^\beta d\tau \int d^N x \bar{\Psi} [-i\gamma_0(\partial_0 + A_0 + i\mu) - i\gamma_i \partial_i - M] \Psi \right\} . \quad (16)$$

As for the case of bosons, one can write the fermionic partition function as a determinant [6, 15]

$$\begin{aligned}\mathcal{Z}_{Fermions} &= \det_D(i\mathcal{D} - M)|_A \\ &= \left[\det(-D^2 + M^2)|_A \right]^{+1},\end{aligned}\quad (17)$$

where \det_D means the calculation over Dirac indices and the subscript A means that the eigenvalues of each operator are computed with antiperiodic boundary conditions. Hence, for the operator $-D^2 + M^2$, the eigenvalues are given by (3), but now the Matsubara frequencies are $\omega_n = (2\pi/\beta)(n + 1/2)$ with $n \in \mathbb{Z}$, so that

$$\lambda_{nk}^F = \left[\frac{(2n+1)\pi}{\beta} + i\mu + A_0 \right]^2 + \vec{k}^2 + M^2, \quad (18)$$

and the corresponding free energy is given by [15]

$$\Omega_F(\beta, \mu, A_0) = 4V \sum_{n=1}^{+\infty} \cos[n(\pi + \beta A_0)] \cosh(n\beta\mu) \left(\frac{M}{2\pi n\beta} \right)^{\frac{N+1}{2}} K_{\frac{1}{2}(N+1)}(n\beta M), \quad (19)$$

which can also be written as

$$\begin{aligned}\Omega_F(\beta, \mu, A_0) = -\frac{TV}{2(2\pi)^N} \int d^N k \Big\{ \ln \left[1 + e^{-2\beta(\omega-\mu)} - 2 \cos(\pi + \beta A_0) e^{-\beta(\omega-\mu)} \right] \\ + (\mu \rightarrow -\mu) \Big\}.\end{aligned}\quad (20)$$

Noting the similarity of Eqs. (19), (20) for the fermionic free energy with Eqs. (6), (12) for the bosonic case we see that the previous analysis for the bosonic scalar field follows immediately with minor changes. Here, the condensation condition is modified to

$$\cosh \beta(\mu - M) = \cos(\pi + \beta A_0), \quad (21)$$

so that we see that fermions can condense when $\mu = M$ analogous to bosons, but the topological gauge field should assume the values

$$A_0' = \frac{(2n+1)\pi}{\beta}, \quad (n \in \mathbb{Z}). \quad (22)$$

Note that these values for A_0' cannot be put to zero by a gauge transformation $\Delta A_0 = 2n\pi/\beta$, because of (anti)periodic boundary conditions of (fermion) gauge field. In this

case in the trivial ($n = 0$) topological sector A'_0 is non vanishing which value π/β we interpret as the exact amount to transmutate the fermion into a boson. The values for A'_0 correspond to the *maxima* of the free energy Eq. (19), which is exactly minus the bosonic free energy Eq. (6). This relation between the free energies of the bosonic and fermionic cases was anticipated by Actor [15], although without discussing BEC. The remaining discussion on condensation is unaltered unless for the substitution $A_0 \rightarrow A'_0$ and the critical temperatures and dimensions are unchanged.

4 Conclusion

We described here a kind of finite temperature bosonization (or fermionization) by including the topological gauge field A_0 in the partition function. We have shown that no condensation occurs for charged scalar fields coupled to a constant and uniform gauge field $(A_0, \vec{0})$ with $A_0\beta \neq 2n\pi$. However, a charged fermionic field interacting with a topological field $A_0 = (2n + 1)\pi/\beta$ will also condense, so this should correspond to a bosonization process. Reversely, adding such a field to a boson we fermionize it once they will exclude rather than condense. This picture is valid only at finite temperatures since at zero temperature ($\beta \rightarrow \infty$) the topological field A_0 which allows BEC vanishes.

This fermionization/bosonization process at finite temperature can also be seen from the A_0 -dependent distribution function corresponding to the bosonic free energy (6) which we write as:

$$f_{A_0}(\beta, \mu) = \Re \left\{ \frac{1}{e^{i\beta A_0} e^{\beta(\omega - \mu)} - 1} \right\}. \quad (23)$$

When $\beta A_0 = 2n\pi$ we obtain the Bose-Einstein distribution function

$$f_0(\beta, \mu) = \frac{1}{e^{\beta(\omega - \mu)} - 1} \quad (24)$$

and when $\beta A_0 = (2n + 1)\pi$ we get

$$f_\pi(\beta, \mu) = -\frac{1}{e^{\beta(\omega - \mu)} + 1}, \quad (25)$$

which is minus the Fermi-Dirac distribution function, so in some sense the boson interacting with $A_0 = (2n + 1)\pi/\beta$ corresponds to an antifermion. Analogously, we can

analyze the case where fermions interact with the topological field A_0 giving rise to the distribution function:

$$f_{A_0}^F(\beta, \mu) = \Re \left\{ \frac{1}{e^{i\beta A_0} e^{\beta(\omega - \mu)} + 1} \right\}. \quad (26)$$

In this case, if $\beta A_0 = 2n\pi$ we find the Fermi-Dirac distribution function, $-f_\pi(\beta, \mu)$, and if $\beta A_0 = (2n + 1)\pi$ the distribution function is minus the Bose-Einstein one, $-f_0(\beta, \mu)$, and the fermion is transmuted into an antiboson. For noninteger values of $\beta A_0/\pi$ these distribution functions may describe interpolating statistics between Fermi-Dirac and Bose-Einstein ones as discussed in [18].

The results of transmutation of fermions into bosons found here for a gas of relativistic charged fermions are similar to those found in [19] within the 2+1 dimensional Gross-Neveu model and in [20] for an $SU(N)$ gauge theory at nonzero quark density where both considered an imaginary chemical potential. From our analysis, we also expect that a similar study of bosonic models at finite temperature with an imaginary chemical potential should present transmutation of bosons into fermions.

Acknowledgments. The authors acknowledge interesting discussions with M. B. Silva Neto and for calling our attention to Refs. [19] and [20]. H.B.-F. thanks the hospitality of MIT-CTP where part of this work was done. H.B.-F. and C.F. were partially supported by CNPq (Brazilian agency).

5 Appendix: High and low temperature limits

Here for self-completeness we show some details on the high and low temperature limits relevant for BEC when the free energy is expressed in terms of Bessel functions, as in Eq. (6). In the ultrarelativistic regime $\beta M \ll 1$ we can use the following property of the Bessel functions

$$\lim_{x \rightarrow 0} K_\nu(x) = \Gamma(\nu) 2^{\nu-1} x^{-\nu}; \quad (\nu > 0) \quad (27)$$

so that the free energy (6) reduces to ($\beta A_0 = 2n\pi$)

$$\Omega(\beta, \mu) = -\frac{2V}{(\beta\sqrt{\pi})^{N+1}} \Gamma\left(\frac{N+1}{2}\right) \sum_{n=1}^{+\infty} \cosh(n\beta\mu) \left(\frac{1}{n}\right)^{N+1}. \quad (28)$$

From the definition of the charge density Eq. (7) we find that at high temperature it is given by

$$\rho(\beta, \mu) = \frac{2}{(\sqrt{\pi})^{N+1}} \left(\frac{1}{\beta}\right)^N \Gamma\left(\frac{N+1}{2}\right) \sum_{n=1}^{+\infty} \sinh(n\beta\mu) \left(\frac{1}{n}\right)^N. \quad (29)$$

From this charge density one can find the Eq. (8).

For the nonrelativistic limit we can use the asymptotic expansion for the Bessel function

$$K_\nu(x) \simeq \sqrt{\frac{\pi}{2x}} e^{-x}, \quad (30)$$

valid when $|x| \gg 1$ and $-\pi/2 < \arg x < \pi/2$. Taking the limit $\beta M \gg 1$ in the free energy(6), we have in this case ($\beta A_0 = 2n\pi$):

$$\Omega(\beta, \mu) = -\frac{2V}{\beta} \left(\frac{M}{2\pi\beta}\right)^{\frac{N}{2}} \sum_{n=1}^{+\infty} \cosh(n\beta\mu) \left(\frac{1}{n}\right)^{1+\frac{N}{2}} e^{-n\beta M}. \quad (31)$$

In this regime the charge density reads

$$\rho(\beta, \mu) = 2 \left(\frac{M}{2\pi\beta}\right)^{\frac{N}{2}} \sum_{n=1}^{+\infty} \sinh(n\beta\mu) \left(\frac{1}{n}\right)^{\frac{N}{2}} e^{-n\beta M}. \quad (32)$$

Exactly as in the ultrarelativistic case, here the critical point is reached when $\mu \rightarrow M$. So, in the nonrelativistic limit, the condensation condition implies that $\beta\mu \gg 1$ and then the charge density reduces to

$$\rho(\beta, \mu) = \left(\frac{M}{2\pi\beta}\right)^{\frac{N}{2}} \sum_{n=1}^{+\infty} \left(\frac{1}{n}\right)^{\frac{N}{2}} e^{n\beta(\mu-M)}, \quad (33)$$

from which we find Eq. (10).

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